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Research Article

A Simple Method to Generate a Stationary Random Process with Arbitrary Desired Probability Density Function and Autocorrelation

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***Abstract*—A stationary random process with a desired Probability Density Function (PDF) and Autocorrelation Function (ACF) is generated in this paper. These two required properties are achieved simultaneously using a simple design procedure. The proposed method consists of a digital filter followed by non-linear function. The digital filter is designed to shape the ACF, while the non-linear function is designed to produce the desired PDF at the final output. A simple polynomial form is used to represent the nonlinear function and a simple non-recursive linear phase digital filter is used. Six scenarios are considered, that combine two types of ACF and three types of PDF. Computer simulation results show that the required properties are achieved with high accuracy using Chi-Square goodness of fit test.**

***Index Terms*— ACF, Correlation shaping filter, Non-Gaussian noise generation, PDF, Stationary random process.**

I. INTRODUCTION

A stationary random process with a desired PDF and autocorrelation properties (or a desired Power Spectral Density PSD) are widely used in modeling, testing, and simulation of Radar receiver [1, 2], under water sonar [3, 4], some types of communication channels [5], Electronic Counter Measures (ECM) and some other physical systems. Most of the previous works in this field are either generating a random process with a desired PDF [6, 7], or generating a random process with a desired autocorrelation properties [8, 9].

The general philosophy for generating a correlated random process with a desired PDF, is to start with a uniformly distributed white process, and to implement filtering in time domain to achieve the desired correlation properties, then a nonlinear transformation is used to shape the required PDF.

Using this approach, few works were introduced to generate both properties at the same time. In 2002, Shen Minfen etal [10] introduced a long-complicated method to generate a non-Gaussian random process with a desired PSD and PDF based on Hermite polynomials and Yale-Walker equations. In 2010, Steven Kay [11] introduced a method to represent and generate a non-Gaussian random process with a desired PSD and a special class of PDF. However, these methods are relatively complicated and there were some conditions imposed on the required types of PDF. In 2012, M. Johnny presents a method to generate a signal with arbitrary PSD and first order PDF [12], only nonlinear transformation function is used to achieve the task. In 2020, Shaowei Dai etal [13] suggested to use particle swarm optimizer, to design the correlation shaping filter and the required nonlinear transformation. This design procedure is believed to be time consuming with no guarantee to reach a convergent optimal solution, for some types of PDF and correlation properties.

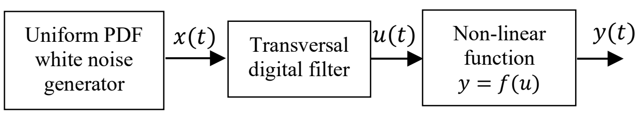
In 2022, Qiang Yi presents nonlinear memoryless transformation method for non-Gaussian clutter model [14]. In 2025, Federica Genovese and Giuseppe Muscolino [15] present a design process for the PSD at the output based on defined PSD at the input, in this case the autocorrelation function is not considered. Another method is presented in 2025 by Federica Genovese and Alessandro Palmeri [16], the method uses circular harmonic Wavelet transform to generate stochastic process of artificial accelerograms for seismic ground motions, the presented method adopts only exponential autocorrelation function.

In this paper, simple design procedures for both the transversal digital filter and the non-linear transformation are suggested. The procedure starts with finding the required nonlinear transformation, then the moment generating function of the Gaussian random process is used to find the impulse response of the transversal digital filter.

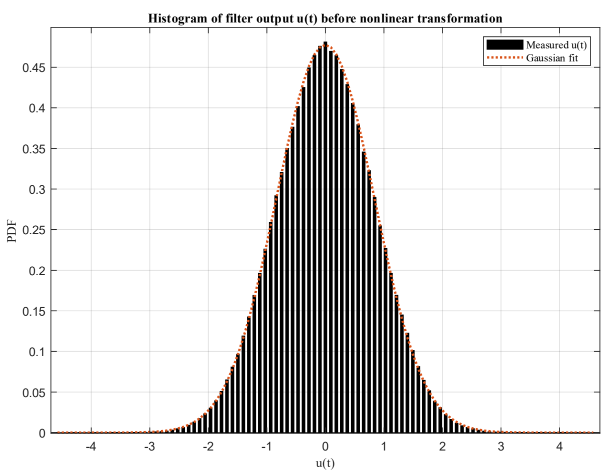
The paper is organized as follows: Section 2 gives the general description of the proposed generator, Section 3 describes the detailed design procedure, Section 4 showed some computer simulation results. Finally, section 5 will give some concluded remarks.

II. General Description of the Proposed Generator

The general block diagram of the random process generator is shown in Figure 1-a. The source of randomness $x(t)$ is obtained from a uniform PDF noise generator with white spectral PSD or Delta Dirac impulse-like autocorrelation properties. This generator is easily implemented by a classical shift register maximal length Pseudo-Noise (PN) generator or some chaotic systems [17, 18]. The output of the N-tap transversal digital filter $u(t)$ has specific autocorrelation properties $R_u(\tau)$ which is different from the impulse like properties of $R_x(\tau)$ and different from the desired autocorrelation properties $R_y(\tau)$ of the generated process $y(t)$. However, and due to central limit theorem, the output of the digital filter $u(t)$ is safely assumed to have Gaussian PDF due to the large number of samples of the weighted values of $x(t)$. This is verified by computing the PDF for the measured $u(t)$ as shown in Figure 1-b. Finally, $u(t)$ is applied to a non-linear function $y = f(u)$ to generate the required PDF of $y(t)$, assuming that $u(t)$ has Gaussian PDF.



(a) General block diagram of the random process generator



(b) PDF of $u(t)$

Fig. 1. a) General block diagram of the random process generator, b) PDF of $u(t)$

III. Proposed Design Procedure

Suppose that the required PDF of $y(t)$ is $p(y)$ and the required autocorrelation function of $y(t)$ is $R_y(\tau)$. Although $p(y)$ represents the statistical properties of $y(t)$ and for stationary processes, it is related to $R_y(\tau)$ by the initial and the final values of $R_y(\tau)$ [6], i.e.,

$$\lim_{\tau \to 0} R_y(\tau) = E\{y^2(t)\} = \sigma_y^2 + \mu_y^2 \quad (1)$$

and

$$\lim_{\tau \to +\infty} R_y(\tau) = \mu_y^2 \quad (2)$$

where σ_y and μ_y are the standard deviation and the mean of the desired process $y(t)$, while E is the expectation operator. For the desired $p(y)$ and $R_y(\tau)$, the impulse response of the digital filter $h(nT_s)$ (T_s is the sampling interval in time domain) and the nonlinearity function $y = f(u)$ must be found. In general, the procedure starts to design the system from its output (the nonlinearity function $y = f(u)$) to its input the impulse response of the digital filter $h(nTs)$. Of course, testing the desired generated process is then done by applying a random process with a uniform PDF and delta shape autocorrelation function to the system shown in Figure 1-a.

Now, the design procedure has the following steps:

Step 1:

The nonlinearity function $y = f(u)$ is expressed in a simple nth order polynomial:

$$y = \sum_{i=0}^{n} a_i \, u^i \quad (3)$$

Next, the PDF of $y(t)$ and $u(t)$ are related by their Cumulative Distribution Function (CDF) as [19]:

$$\int_{-\infty}^{y} p(y)dy = \int_{-\infty}^{u} p(u)du \quad (4)$$

using Eq. 4, different values of u are found for different values of y either analytically or numerically (u is safely assumed to have Gaussian PDF due to the central limit theorem and the large number of filter coefficients). A table of y versus u values is obtained from which, and using any simple curve fitting, the coefficients of the nonlinearity (a_0, a_1, a_2,…, a_n) can be obtained.

Step 2:

The autocorrelation $R_u(\tau)$ is found in terms of the desired autocorrelation $R_y(\tau)$ and the coefficients (a_0, a_1, a_2,…, a_n). This is done as follows:

$$R_y(\tau) = E\{y(t).\, y(t+\tau)\}$$

Using Eq. 3 then:

$$R_y(\tau) = E\{\left(\sum_{i=0}^{n} a_i \, u_1^i\right) \left(\sum_{i=0}^{n} a_i \, u_2^i\right)\}$$

$$R_y(\tau) = E\{(a_0 + a_1 u_1 + a_2 u_1^2 + \cdots + a_n u_1^n)(a_0 + a_1 u_2 + a_2 u_2^2 + \cdots + a_n u_2^n)\} \quad (5)$$

The above equation has $(n+1)^2$ terms of the form:

$$E\{a_k \, a_m \, u_1^k \, u_2^m\} \text{ for } k = 0, 1, 2, \ldots, n \text{ and } m = 0, 1, 2 \ldots, n$$

and since a_k and a_m are constants found from step 1, then:

$$E\{a_k \, a_m \, u_1^k \, u_2^m\} = a_k a_m \, E\{u_1^k \, u_2^m\}$$

The joint moments $E\{ u_1^k \, u_2^m\}$ of the random variables u_1 and u_2 can be obtained from the joint characteristic function through successive differentiation. Since u_1 and u_2 are assumed to be Gaussian random variables, then their joint characteristic function is given by [19] :

$$M\{ju_1, ju_2\} = e^{\{-0.5(Su_1^2 + 2S\rho u_1 u_2 + Su_2^2)\}} \quad (6)$$

Where:

$S = \sigma_u^2 = R_u(0)$ for zero mean $u(t)$ and

ρ = correlation coefficient of

$$u(t) = \frac{R_u(\tau)}{R_u(0)} = \frac{R_u(\tau)}{S}$$

The joint moments $E\{u_1^k\, u_2^m\}$ is calculated as [19]:

$$E\{u_1^k\, u_2^m\} = \lim_{\substack{u_1 \to 0 \\ u_2 \to 0}} (-j)^{k+m} \frac{\partial^{k+m}\, M(ju_1, ju_2)}{\partial^k u_1\; \partial^m u_2} \quad (7)$$

MATLAB m-file code with symbolic operations is used to find the joint moment matrix $[B]$ for any desired value of n. The results can be arranged in a matrix $[B]$. As an example, and for $n = 5$ (5th order nonlinear polynomial function $f(u)$), the matrix $[B]$ is found to be:

$$[B] = \begin{bmatrix} 1 & 0 & S & 0 & 3S^2 & 0 \\ 0 & S\rho & 0 & 3S^2\rho & 0 & 15S^3\rho \\ S & 0 & S^2 + 2S^2\rho^2 & 0 & 12S^3\rho^2 + 3S^3 & 0 \\ 0 & 3S^2\rho & 0 & 6S^3\rho^3 + 9S^3\rho & 0 & 60S^4\rho^3 + 45S^4\rho \\ 3S^2 & 0 & 12S^3\rho^2 + 3S^3 & 0 & 24S^4\rho^4 + 72S^4\rho^2 + 9S^4 & 0 \\ 0 & 15S^3\rho & 0 & 60S^4\rho^3 + 45S^4\rho & 0 & 120S^5\rho^5 + 600S^5\rho^3 + 225S^5\rho \end{bmatrix}$$

This matrix is then used to find any joint moment $E\{u_1^k\, u_2^m\}$ for any $k = 0, 1, 2, \dots, n$ and $m = 0, 1, 2 \dots, n$. For example: $E\{u_1^3\, u_2^5\} = 60S^4\rho^3 + 45S^4\rho$ and so on.

Also, one can note that:

$i)$ $E\{u_1^k\, u_2^m\} = 0$ $\;if\;(k + m)$ is odd and

$ii)$ $E\{u_1^k\, u_2^m\} = E\{u_1^m\, u_2^k\}$ i.e. $[B]$ is symmetric.

Now, expanding Eq. 5 for $n = 5$ (say) and using the elements of $[B]$ to find $E\{u_1^k\, u_2^m\}$, then:

$$\begin{aligned} R_y(\tau) = a_0^2 + a_1^2 R_u(\tau) + a_2^2[R_u^2(0) + 2R_u^2(\tau)] \\ + a_3^2[6R_u^3(\tau) + 9R_u^2(0)R_u(\tau)] \\ + a_4^2[24R_u^4(\tau) + 72R_u^2(0)R_u^2(\tau) + 9R_u^4(0)] \\ + a_5^2[120R_u^5(\tau) + 600R_u^2(0)R_u^3(\tau) \\ + 225R_u^4(0)R_u(\tau)] + 2a_0a_2R_u(0) \\ + 6a_0a_4R_u^2(0) + 6a_1a_3R_u(0)R_u(\tau) \\ + 30a_1a_5R_u^2(0)R_u(\tau) \\ + a_2a_4[24\, R_u^2(\tau)R_u(0) + 6R_u^3(0)] \\ + a_3a_5[72R_u(0)R_u^3(\tau) + 90R_u^3(0)R_u(\tau) \\ + 48R_u^2(0)R_u^2(\tau)] \end{aligned} \quad (8)$$

Where S is replaced by $R_u(0)$ and each $(S\rho)$ is replaced by $R_u(\tau)$.

The coefficients $(a_0, a_1, a_2, \dots, a_5)$ in Eq. 8 are known from step-1. Eq. 8 can now be used to find $R_u(\tau)$ for a given desired $R_y(\tau)$. To do so, $R_u(0)$ must be found first since it is included in Eq. 8 as a constant. To find $R_u(0)$, put $\tau = 0$ in Eq. 8 and re-arrange terms to have:

$$R_y(0) = a_0^2 + R_u(0)[a_1^2 + 2a_0a_2] + R_u^2(0)[3a_2^2 + 6a_0a_4 + 6a_1a_3] + R_u^3(0)[15a_3^2 + 30a_1a_5 + 30a_2a_4] + R_u^4(0)[105a_4^2 + 210a_3a_5] + R_u^5(0)[945a_5^2] \quad (9)$$

Now, since $R_y(0)$ is known, Eq. 9 can be solved for $R_u(0)$ using Newton's method, then substituted again in Eq. 8 to get:

$$\textstyle\sum_{i=0}^{n} C_i\, R_u^i(\tau) = 0 \quad (10)$$

For $n = 5$, the coefficients C_i are:

$$C_0 = a_0^2 + a_2^2\, R_u^2(0) + 9a_4^2\, R_u^4(0) + 2\, a_0a_2\, R_u(0) + 6\, a_0a_4\, R_u^4(0) + 90\, a_3a_5R_u^3(0) - R_y(\tau)$$

$$C_1 = a_1^2 + 9a_3^2R_u^2(0) + 225a_5^2R_u^4(0) + 6a_1a_3R_u(0) + 30\, a_1a_5R_u^2(0) + 90a_3a_5R_u^3(0)$$

$$C_2 = 2a_2^2 + 72a_4^2\, R_u^2(0) + 24a_2a_4R_u(0) + 48a_3a_5R_u^2(0)$$

$$C_3 = 6a_3^2 + 600a_5^2 + 72a_3a_5R_u(0)$$

$$C_4 = 24a_4^2$$

$$C_5 = 120a_5^2$$

Eq. 10 is solved for different values of τ using Newton's method and for discrete values of $\tau = LT_s, L = 0, 1, 2, \dots, N - 1$, where:

T_s is the sampling interval in time domain.

N is the number of samples taken from the desired correlation function $Ry(\tau)$.

As a conclusion for step-2, Eq. 10 is solved for N-samples of $R_u(LT_s)$ from the corresponding values of N samples of $R_y(LT_s)$.

Step 3:

Finally, the problem is reduced into the design of a digital filter, that shapes the correlation of $x(t)$ (delta shape) into the calculated $R_u(LT_s)$ obtained from step-2. In general:

$$\emptyset_u(Lf_s) = |H(jLf_s)|^2 \emptyset_x(Lf_s) \quad (11)$$

Where $H(jLf_s)$ is the filter transfer function and f_s is the sampling interval in the frequency domain. $\emptyset_x(Lf_s)$ and $\emptyset_u(Lf_s)$ are the input and output power spectral densities of the filter.

The amplitude characteristics of the required digital filter can be found from Eq. 11 as:

$$|H(jLf_s)| = \sqrt{\frac{\emptyset_u(Lf_s)}{\emptyset_x(Lf_s)}}$$

Assuming a delta function correlation of $x(t)$, then $\emptyset_x(Lf_s) = 1$ (normalized). Hence:

$$|H(jLf_s)| = \sqrt{\emptyset_u(Lf_s)} = \sqrt{\mathfrak{F}(R_u(LT_s)} \quad (12)$$

Where Ƒ is the discrete Fourier transform.

If a Finite Impulse Response (FIR) digital filter is used, then the impulse response of such a filter is found from its amplitude characteristics given in Eq. 12. In fact, there are infinite number of filters that satisfy Eq. 12, depending on the infinite number of ways to choose the phase characteristics. One of the simplest ways is to use the linear phase characteristics, that has a real symmetrical impulse response [20]:

$$H(LT_s) = h[(2N-1-L)T_s] \tag{13}$$

for $L = 0, 1, 2, \dots, N-1$

The impulse response of this filter is determined by:

$$h(LT_s) = \frac{1}{2N}\left[|H(0)| + 2\sum_{i=1}^{N-1}(-1)^i\,|H(jLf_s)|\,cos\left(\frac{\pi\, i(2L+1)}{2N}\right)\right] \tag{14}$$

for $L = 0, 1, 2, \dots, N-1$

IV. Simulation Results

In this section, the simulation results of the proposed method are addressed. As mentioned in the previous two sections, after generating the variable $x(t)$ at the input or the proposed system as shown in Figure 1-a, the design process begins with setting a target PDF and an ACF for the output $y(t)$. In this paper, six different scenarios of different PDFs and ACFs are explored, to validate the robustness of the proposed method. Table I addresses these scenarios, where the ACFs are:

$$R_{Exp}(\tau) = e^{-4\tau} \tag{15}$$

$$R_{ExpCos}(\tau) = e^{-3\tau}\cos(12\tau) \tag{16}$$

and the custom shape PDF is:

$$p_{custom}(y) = \begin{cases} \frac{2.6}{67.32}[1+\cos(0.4(y+\pi))] & -\frac{7\pi}{2} < y < -\pi \\ \frac{1}{67.32}[4.2-\cos(y)] & -\pi \le y \le \pi \\ \frac{2.6}{67.32}[1+\cos(0.4(y-\pi))] & \pi < y < \frac{7\pi}{2} \end{cases} \tag{17}$$

TABLE I

Target PDF-ACF scenarios

Scenario	ACF	PDF
1	R_{Exp}	Rayleigh
2	R_{ExpCos}	Rayleigh
3	R_{Exp}	Laplacian
4	R_{ExpCos}	Laplacian
5	R_{Exp}	p_{custom}
6	R_{ExpCos}	p_{custom}

In these six scenarios, and to ensure a correct representation of each target PDFs at the system output, the order of the nonlinear function in Eq. 3 is set to 10 for the Rayleigh PDF and set to 20 for the p_{custom} and the Laplacian PDFs. Figures (2-7) present the ACF and the PDF at the output of the proposed system for the six scenarios listed in Table I.

It can be noticed from these figures that the proposed system provides a great fit for the target PDF and ACF, regardless of the different combinations. To provide an objective measure of goodness of fit, the Chi-Square test is considered in this paper. Table II shows the Chi-Square test results for the three types of PDFs which are considered in this paper. The test is applied with degree of freedom of 20 and p-value greater or equal to 5% confidence level. The ACF at the output of the proposed method shows almost exact fit to the target ACF as illustrated in Figures (2-7), which obviates the need to mean square error calculations.

TABLE II

Chi-Square test results for different PDF scenarios.

Scenario	PDF	Chi-Square Statistics	P-value
1	Laplacian	0.0012	1.000
2	Rayleigh	0.0000	1.000
3	p_{custom}	0.0052	1.000

It is clear from Table II that the obtained distributions are highly correlated to the target PDFs, showing that the proposed method produces almost the exact target ACF and PDF in each of the chosen six scenarios.

V. Conclusion

This paper presents a simple yet effective method to produce a stationary random process with desired statistical properties. The proposed method sets two statistical properties, the ACF and the PDF. A digital filter is followed by a nonlinear function are used to achieve this task. The digital filter is designed to provide the ACF properties at the output of the system, while the nonlinear function role is to model the desired PDF distribution. Two types of ACF are considered here, R_{Exp} and R_{ExpCos} as described in equations 15 and 16 respectively. Three types of PDF distributions are investigated, Laplacian, Rayleigh and p_{custom} as described in Eq. 17. To evaluate the proposed method, six scenarios that represent different combinations of the ACFs and PDFs, are assumed. The simulation results show high correlation between the target statistical properties at the output of the proposed system and the target ACF and PDF of each scenario, as shown in Table 2.

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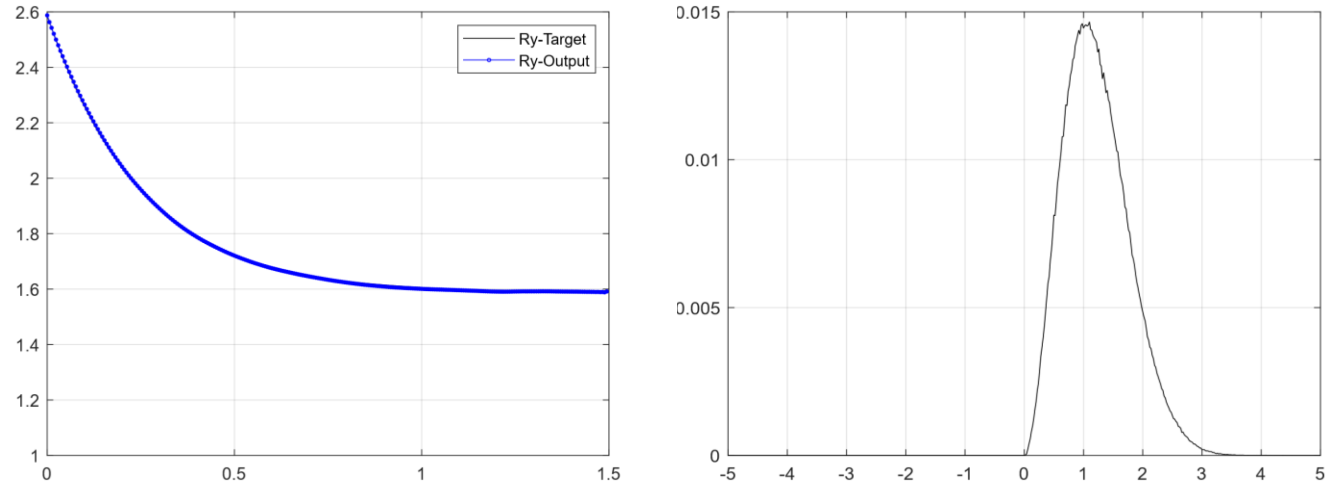
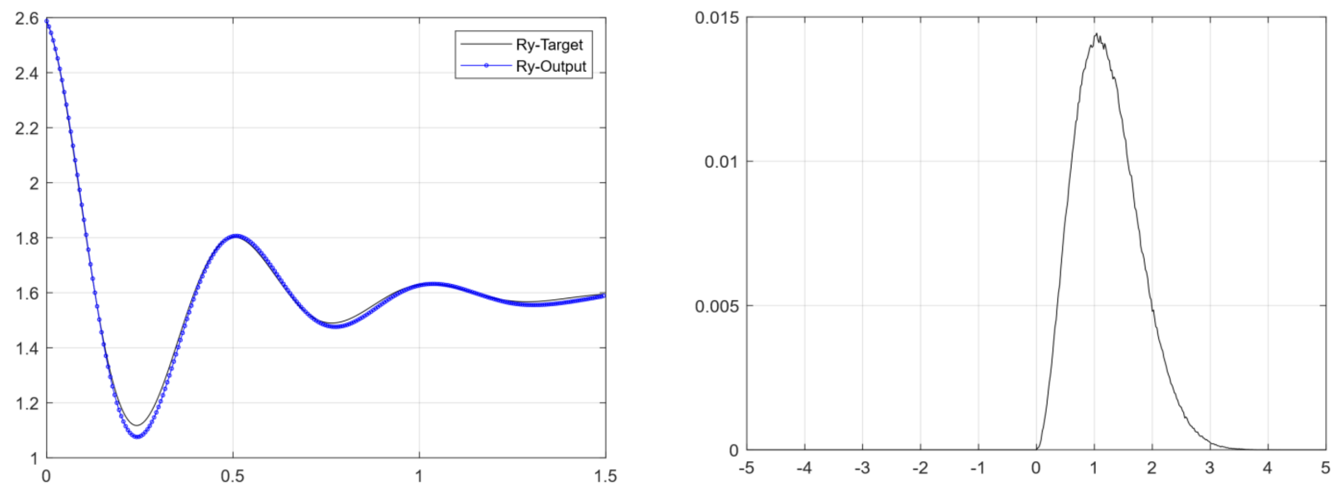


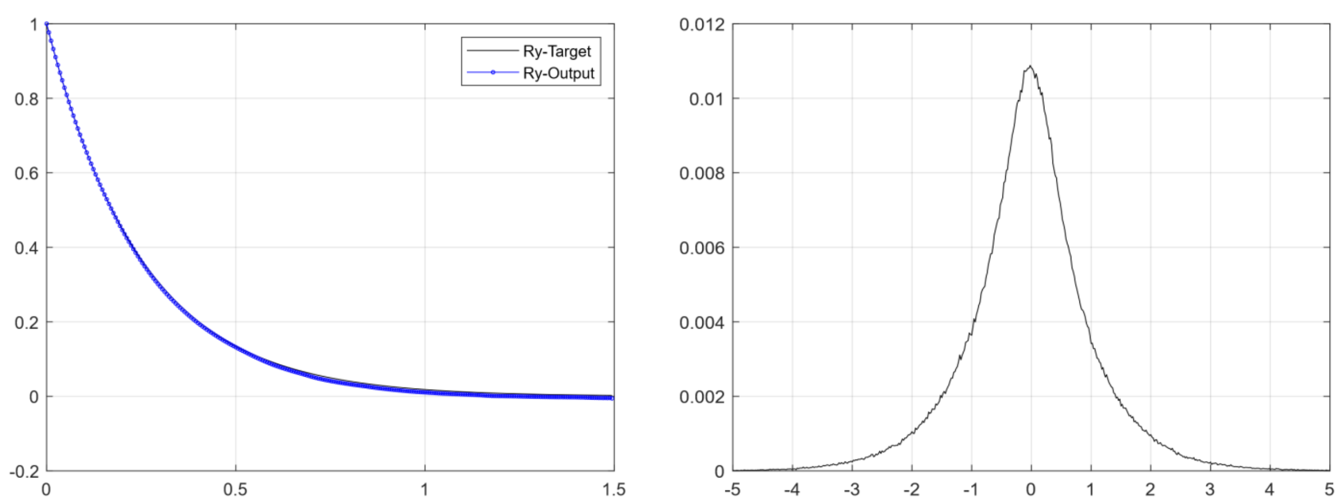
(a) ACF (b) PDF

Fig. 2. Output ACF and PDF for Scenario 1.



(a) ACF (b) PDF

Fig. 3. Output ACF and PDF for Scenario 2.



(a) ACF (b) PDF

Fig. 4. Output ACF and PDF for Scenario 3.

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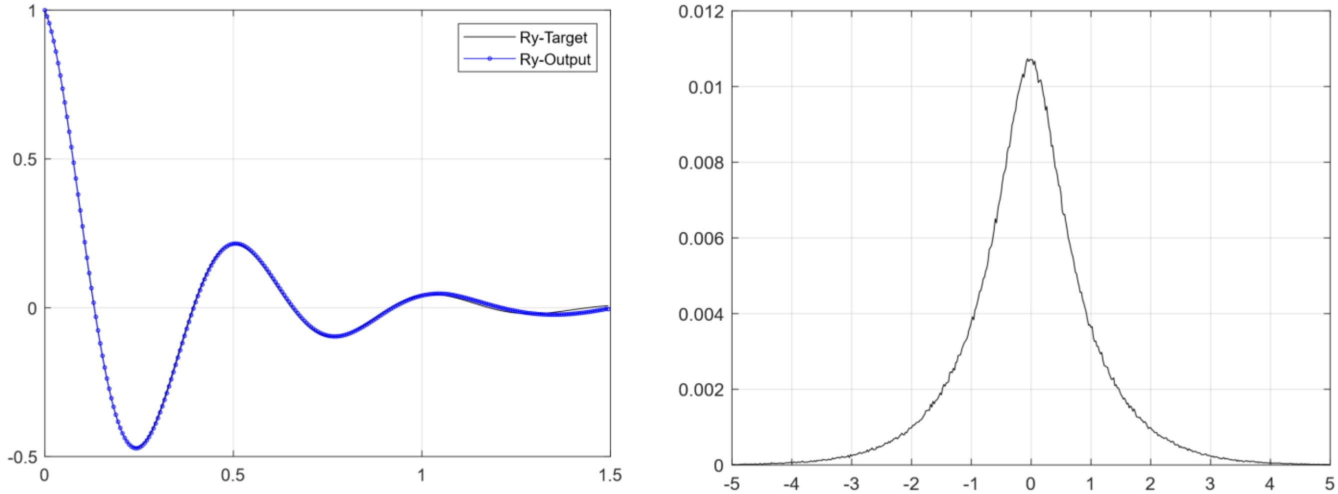
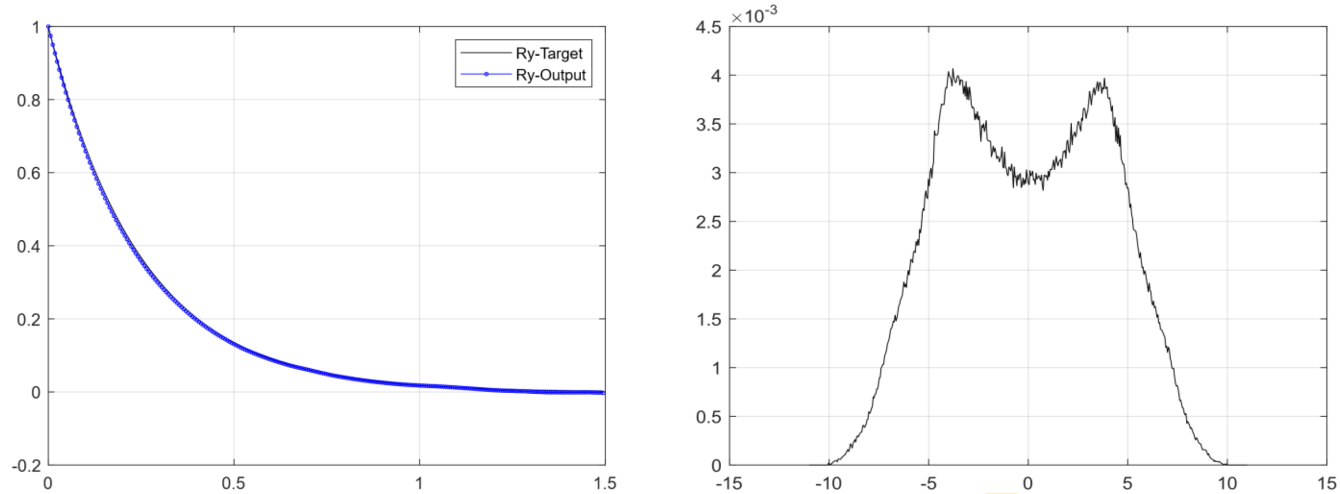


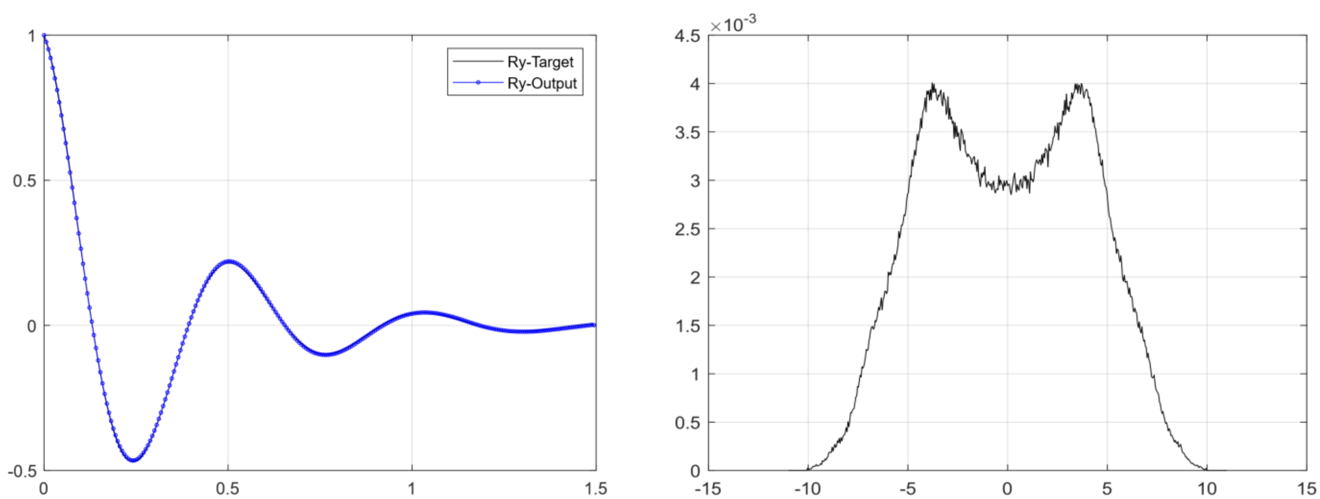
(a) ACF (b) PDF

Fig. 5. Output ACF and PDF for Scenario 4.



(a) ACF (b) PDF

Fig. 6. Output ACF and PDF for Scenario 5.



(a) ACF (b) PDF

Fig. 7. Output ACF and PDF for Scenario 6.